Architecture independent vertical TPU estimation for multi-beam sonar processing

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Abstract. The CUBE (Combined Uncertainty and Bathymetry Estimator) algorithm[1] is commonly used to establish a surface of likely depth from the point-cloud type data produced by multi-beam echo-sounders. Two fundamental inputs to the CUBE algorithm are horizontal and vertical components of Total Propagated Uncertainty (TPU)[2]. The algorithm requires that TPU values closely match the performance of the sonar system. Mismatch between TPU and the actual sonar performance can cause inappropriate weighting of soundings and may result in a calculated surface that is not the best estimate of depth. Typical TPU values used for CUBE are comprised of a-priori vertical and horizontal uncertainty estimates. This a-priori estimate of TPU *should* compare well to the real-world performance of a particular system. However, the TPU models are not transferable between systems with fundamentally different designs. Furthermore, a-priori calculations do not account for performance degradation of systems in sub-optimal conditions or over time. This paper proposes a methodology – the "maximum-median method" – to estimate representative vertical TPU values for each beam that may be used to enhance CUBE algorithm performance. The estimation uses analysis of the median and maximum variance across sets of consecutive pings. This method produces representative vertical TPU values with the advantage of down-weighting noisy beams that are not performing to their a-priori estimates.

1. INTRODUCTION

The use of single-beam sonar for bathymetric surveying produces depth measurements at discrete times and locations with little data overlap. The accuracy of a set of depth measurements (soundings) is easily assessed using horizontal and vertical a-priori error models, combined with a subjective assessment by an experienced surveyor. Finally, a cross-line comparison is conducted to validate the result. Discrete data such as this can answer the questions: *"at position A, the measured depth was B, is this valid? What was the uncertainty in the measurement?"*

Conversely, multi-beam systems are capable of recording thousands of measurements a second with considerable data redundancy. This requires a different approach to querying the data. Multi-beam data may be treated as a dense point cloud rather than single discrete measurements at discrete positions. From this data a surveyor may ask: *"given this set of measurements, what is the most likely depth? How sure are we of this result?"*

In order to answer these questions efficiently, a large volume of multi-beam soundings must be subjected to a degree of automated analysis. One such method of analysis is to generate a surface from the data with the Combined Uncertainty and Bathymetric Estimation (CUBE) algorithm[1]. This algorithm uses Bayesian estimation, redundancy of measurements and typically a-priori estimates of horizontal and vertical Total Propagated Uncertainty (TPU) to hypothesize the likely depth at a given grid node. It also supplies an estimate of the vertical uncertainty for that hypothesis[2].

Using CUBE to assist in the processing of data has been shown to achieve around a third of the time-cost compared to a purely subjective assessment by a surveyor[3]. This makes the CUBE process an attractive proposition for data processing. However, if the horizontal and vertical TPU models, used as inputs to CUBE, are not reflective of the real performance of a sonar system, then the performance of the algorithm is reduced. In this case, the corresponding gridded data may not represent the best estimate of the bathymetry[4].

Consequently, considerable effort is applied to rigorous modeling of a-priori uncertainty associated with the measurements and equipment performance at each stage of the data collection process[5].

Despite this effort, mathematical modeling can only give an accurate estimate of the uncertainty associated with a system if each component is performing within the manufacturer's specifications and under the conditions over which the model is defined.

This paper examines the assumptions and behavior of typical mathematical models for vertical TPU in the context of CUBE. It then proposes and compares an alternative means for estimating vertical TPU via observation, using the median and maximum variance of a moving window of soundings. This method is referred to herein as the "maximum-median" method.

2. BACKGROUND

Multi-beam echo-sounders are capable of producing exceptionally large volumes of data with high redundancy. The Reson SeaBat 7125, for example, can operate at a pulse repetition rate of 50 pulses per second, collecting 512 data points per pulse for a total of 25,600 depth measurements per second[6]. This translates to about 170 depth measurements per square meter of seabed for a survey vessel moving at 8 knots in 10 meters of water with a swath width multiplier of 3.5 × depth. Subjective examination of this large volume of data would result in significant time-cost for the surveyor. It is also difficult to visualize and disambiguate dense point cloud data effectively. As such, large scale subjective data assessment may introduce a significant perceptual human error component in the determination of depth.

Fortunately, the existence of redundant data allows for efficient methods of statistical analysis, simplifying the process of data inspection with dramatically reduced time-cost[4]. CUBE is one such method that is widely used due to its incorporation into commercially available software, including CARIS[7] and Fledermaus[8] products.

2.1. The CUBE process

The CUBE algorithm processes soundings through three primary stages in order to sequentially update gridded datasets of likely depth, uncertainty and alternate depth hypotheses for ambiguous nodes.

These stages as described by Calder and Wells[2] are:

1. assimilation,
2. intervention, and
3. disambiguation.

Soundings are assimilated against a network of nodes established over the area of interest. The nodal positions are without any horizontal uncertainty as they have been placed systematically. At each nodal position, there is a hypothesis, or multiple hypotheses about the depth at the node. The relative strengths of the hypotheses are adjusted sequentially as soundings are assimilated to the node.

Consequently, the horizontal and vertical TPU associated with each sounding must be converted into just vertical uncertainty relevant to the hypotheses at the nodal position. In order to achieve this, as each sounding is assimilated, its vertical TPU is increased as a function of the squared distance to the node, r^2. Then, the horizontal TPU is also converted into a vertical uncertainty with respect to the first power distance from the node, r. The sounding is then compared to the existing depth hypotheses and uncertainty regions within the node.

If a sounding is not statistically compatible with the existing hypotheses, then an alternative hypothesis is formed during intervention, against which other soundings with similar properties may be assimilated.

Finally, a process of disambiguation occurs where the most likely hypotheses are selected through three tests. First, is the numerical strength of supporting soundings, referred to by Calder and Wells as a "popularity contest". Second, is a local consistency test (against nearest neighboring nodes). Third, is an external consistency test (against some external representation or lower resolution surface).

During this process, the horizontal and vertical TPU values associated with a sounding determine its effect on the support for existing hypotheses, creation of new hypotheses and subsequently, the selection of likely hypotheses. In the default configuration, the vertical TPU contribution to node uncertainty is r times larger than the contribution due to horizontal TPU. Thus, the vertical TPU model used dominates the weighting of soundings and the corresponding selection of hypotheses.

2.2. Typical vertical TPU models

The most widely adopted base model of vertical TPU is that prescribed by Hare[5] in the mid 1990's for use with the Canadian Hydrographic Service beam-forming sonars. The TPU model of Hare describes the vertical component as the combination of sounder measurement variance, beam-opening angle variance, roll, pitch, heave variances and refraction variance. If we consider only the beam-opening angle and sounder measurement variances in this model, i.e. neglecting dynamic tidal, motion or refraction components, then the characteristic distribution of vertical TPU across a beam-forming sonar swath is given by (i):

$$\sigma_d = \sqrt{\cos\theta^2\sigma_r^2 + y^2\sigma_\theta^2 + \left(d\left[1 - \cos\left(\frac{\psi}{2}\right)\right]\right)^2}$$

(i)

where θ is the beam-angle, the variance terms correspond to range and beam-angle variance respectively, d is depth, and ψ is the beam opening angle. The minimum of this equation occurs at the center beam of the swath, where (if no motion) both $y = 0$ and $\theta = 0$. This results in the typical distribution shape shown in figure 1.

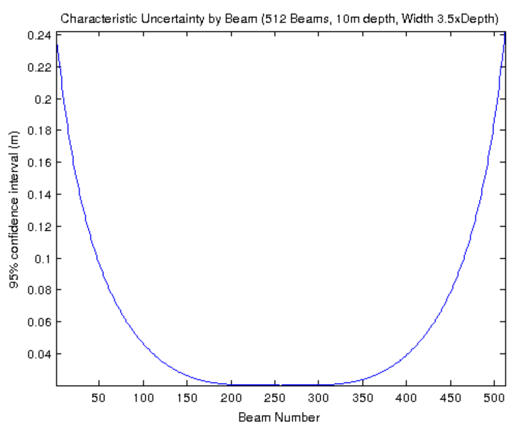

Figure 1: Characteristic distribution of vertical TPU for a typical beam-forming sonar, 10m depth 350% coverage.

It is important to note that whilst beam-forming sonars are common, this model does not adequately describe the vertical TPU associated with sweep systems or phase-differencing bathymetric side-scan systems. The acoustic fundamentals of these systems are markedly different and uncertainty must be calculated accordingly.

In the case of simple sweep systems (i.e. where no phase-differencing is conducted), the transducers are essentially horizontally spaced arrays of single-beam transducers. Consequently, the uncertainty calculation across the swath is a trivial extension of that used for single-beam sonars. The dominating factor describing uncertainty differences between beams is the measurement of lever-arms from the vessel origin to each transducer. The acoustic properties of each transducer should be nearly identical and the error associated with ray-tracing is greatly reduced if all of the transducers are close to vertical.

Phase differencing bathymetric side-scan systems pose a more complicated problem. A bathymetric side-scan system does not necessarily form more than two beams. Some bathymetric side-scan sonars use a small number of formed beams for first angle detection, others may use the interferometric amplitude beam-pattern for first angle detection. In any case, the return of a pulse is post-processed using phase-difference to determine the best estimate of the angle of the return. This results in a very complex and dynamic vertical TPU model associated with the acoustic properties of the echo-sounder. According to Lurton[9], additional factors that must be considered in a phase-differencing vertical TPU model include:

1. signal-to-noise ratio (SNR) in the backscattered signal, causing phase error;

2. baseline decorrelation (especially in Synthetic Aperture Sonar);

3. phase ambiguity due to *modulo* 2π measurement;

4. shifting footprint effect;

5. off-axis signal degradation;

6. signal envelope shape; and

7. nadir (specular) region phase-differencing limitations.

TPU for bathymetric side-scans may be minimized by varying the transducer orientation, shape, spacing and acoustic signal to best match the acoustic properties of the transducers. This leads to a wider range of variables in modeling the TPU associated with bathymetric side-scan systems.

One simplified model of vertical TPU in a phase-differencing system is provided by Torstein[10] who considers that the angular resolution of the array is fixed, making depth uncertainty essentially a function of range and time-delay estimation (ii),

$$\sigma_z = \frac{r}{D}\frac{\cos(\Phi + \Phi_0)}{\cos\Phi}\sigma_\tau \quad \text{(ii)}$$

where σ_τ is the standard deviation of the time delay estimate, Φ is the depression angle in the frame of the transducer body, Φ_0 is the initial tilt of the transducer from vertical, r is the slant range, and D is the transducer baseline. In this case, the σ_τ is a function of the phase difference. As the phase SNR decreases, such as in specular areas (near the nadir) the probability distribution of the time-delay estimate widens. As a result, the σ_τ increases toward the nadir and this term begins to dominate. This manifests as a significant degradation of accuracy for a phase-differencing sonar in the nadir region.

A characteristic a-priori vertical TPU distribution for a phase-differencing sonar (ATLAS Fansweep 20, shallow water version) is shown in figure 2.

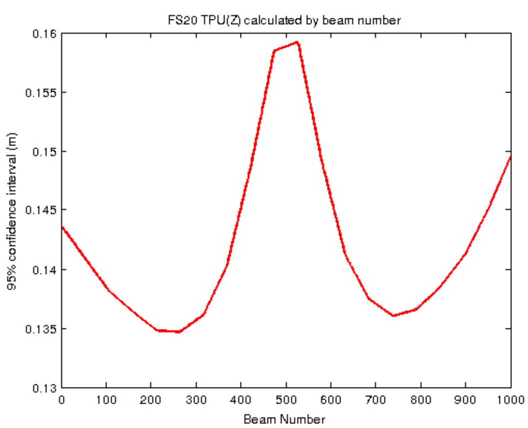

Figure 2: ATLAS Fansweep 20 a-priori vertical TPU distribution. 14m depth, 1000 beams, 1000% depth coverage.

The characteristic distributions shown in figure 1 and 2 demonstrate the difference between the two predominant multi-beam echo-sounder types. Figure 1 shows that the beam-forming sonar typically has its most accurate soundings at the center of the swath. Conversely, in figure 2, the phase-differencing sonar has a significantly lower accuracy in the center but better accuracy in the region normal to the transducer (the transducer faces in this example are mounted at 60º inclines). This higher accuracy region extends the coverage (in this example) of the interferometric system to over three times that of a single-head beam-forming system.

It follows that there cannot be a "one size fits all" mathematical model for vertical TPU. Moreover, the substitution of one model for another when used in the CUBE process can cause soundings to be weighted inappropriately, diminishing the performance of CUBE. This has been demonstrated in a coarse example[4] using cross-line comparison with CUBE surfaces of bathymetric side-scan data processed with manufacturer's vertical TPU estimates, and with TPU modeled by the Hare method.

Additionally, the mathematical models have another significant dilemma. They do not adequately capture the uncertainty attributable to under-performing systems, or systems being employed in sub-optimal conditions. For example, the SNR directly impacts the ability for a bathymetric side-scan sonar to resolve phase-difference. Consequently, when operating subject to increased flow noise or in heavy rain, for example, the ability of a bathymetric side-scan sonar to determine range and beam-angle is diminished.

There are many more examples of situations that cause higher uncertainty in some beams that would not be captured by the mathematical models. Some of these situations affecting both phase-differencing and beam-forming systems include:

1. thermal circuit noise,
2. reverberation,
3. growth of marine life on the transducer faces, and
4. reduced gain from failing transducer elements.

The fundamental design of the CUBE algorithm should cause it to flag evidence of these problems by showing high nodal uncertainty values, or high hypothesis counts. However, the CUBE process still relies on an accurate assessment of a-priori TPU to reliably select the best hypotheses.

In order to gain an estimate of shifting uncertainty due to such conditions and to assist CUBE in data cleaning, empirical uncertainty data is required.

3. VERTICAL TPU ESTIMATION FROM COLLECTED SOUNDINGS

The proposed method for vertical TPU estimation relies on statistical treatment of sequential pings. It requires significant redundancy of data and several key assumptions must be met.

In order to apply this method appropriately, these assumptions and their potential impact on uncertainty estimation must be considered. It is also important to note that this methodology produces uncertainty values that are representative and useful for weighting soundings in the CUBE process, but the estimates are not exact.

3.1. Key Assumptions

The proposed method is built upon several key assumptions as follows:

1) The nature of the seabed is locally homogeneous for short along-track distances

For short along-track distances, the variance of the sea-floor topography is expected to be small, and generally consistent across the swath. Rapid change of variance between beams is then assumed to be due to beam performance differences, rather than sea-floor undulation. This assumption will not hold for very dynamic seabeds or soundings over significant features.

2) Each beam is independent

This assumption is not strictly true, especially in the case of a bathymetric side-scan sonar which measures correlation to form many soundings from each beam[11]. However, for the purpose of this method, the term "beams" is used to describe the final set of discrete recorded measurements across the swath which are assumed to be independent from one-another. Any correlation is neglected.

3) The contribution to observed vertical depth variance due to horizontal TPU is less than that due to sea floor topography

As we are considering a small along-track distance and a locally homogeneous seabed, any *horizontal* variance will increase the observed *vertical* variance by some small value, less than the amount of variance due to sea-floor topography. The CUBE order-*r* propagation of modeled horizontal TPU should be sufficient to capture an estimate of the contribution of horizontal uncertainty to vertical uncertainty at the node.

4) The contribution of tidal uncertainty to vertical TPU is independent of measurement uncertainty by the echo-sounder.

Uncertainty in the tidal model is not a significant contributor to measurement variance over a small time-frame, and thus over a short along-track distance. As such, it cannot be assessed in real-time. Unless conducting an ellipsoid referenced survey, tidal uncertainty still needs to be propagated, but it must be modeled separately to echo-sounder uncertainty.

3.2. Estimating vertical TPU by the maximum-median method

The variance of the topography of the sea floor is not dependent on its measurement. So, it follows that the total observed variance of measurement values obtained from a beam, σ_T^2, over p pings, is the sum of the variance of the sea floor topography, σ_S^2, and the variance of the measurement by a particular beam, σ_B^2:

$$\sigma_T^2 \mid_p = \sigma_S^2 \mid_p + \sigma_B^2 \mid_p \qquad \text{(iii)}$$

In light of assumption 2, assuming each beam is independent, treating all errors as stochastic in nature, the total variance of depth measurements, z, associated with a particular beam, over a set of observations, p, (bounded by ping numbers $p_1 < p < p_2$) is given by:

$$\sigma_T^2 \mid_p = \frac{1}{(p_2 - p_1)} \sum_{i=p_1}^{p_2} \left(z_i - \mu_p\right)^2 \qquad \text{(iv)}$$

The variance associated with the measurement itself, is then given by rearranging (iii) and (iv):

$$\sigma_B^2 \mid_p = \left(\frac{1}{(p_2 - p_1)} \sum_{i=p_1}^{p_2} (z_i - \mu_p)^2 \right) - \sigma_S^2 \mid_p \quad \text{(v)}$$

The problem now is to remove a reasonable estimate of the variance contribution from the sea floor topography, in order to derive an estimate of the variance of the measurement itself.

In light of assumption 1 that the nature of the seabed is locally homogeneous over a short along-track distance, then by median filtering the variances across the swath, described by function $\tilde{x}(\sigma_T^2)$, we can remove "noise" from measurement variation. In doing so, we can obtain an estimate of the variance due to the topography, for each beam, at a reduced resolution from the original data. Obtaining a moving maximum over the swath, described by function $\vee(\sigma_T^2)$, is indicative of the total variance at that reduced resolution. So, the difference between the two functions gives an estimate of the variance of a beam (the "noise"), as a function of the total observed variance of depth, over p pings:

$$\sigma_B^2 \mid_p = \vee\left(\sigma_T^2 \mid_p\right) - \tilde{x}\left(\sigma_T^2 \mid_p\right) \qquad \text{(vi)}$$

Further explanation of the steps taken to compute $\vee(\sigma_T^2)$ and $\tilde{x}(\sigma_T^2)$ is included in appendix A.

Finally, the estimate of vertical TPU (at 95% confidence) for each beam over the window of pings is then given by:

$$TPU_z(B) = 1.96 \times \sqrt{\sigma_B^2 \mid_p} \qquad \text{(vii)}$$

4. RESULTS AND DISCUSSION

4.1. Comparison of maximum-median method and a-priori model performance

All graphical results for estimated vertical TPU are shown at the 95% confidence interval in order to compare the scale of vertical uncertainty against IHO Special Publication S-44[12] limits. For input into CUBE, 68% confidence interval values are usually required.

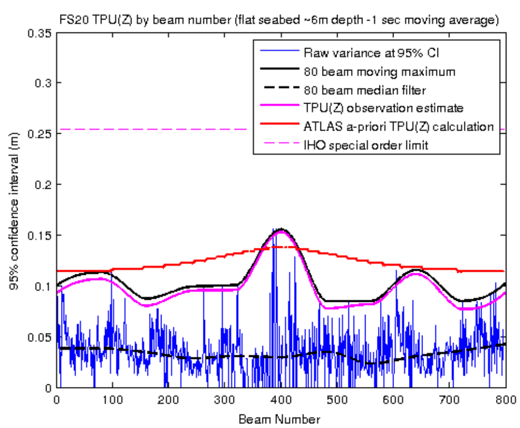

Figure 3: ATLAS FS20 maximum-median estimated (magenta) and a-priori calculated (red) uncertainty. Consistent seabed in optimal conditions. 1 second along-track averaging, 80 beam across track filtering.

Figure 3 shows the a-priori and maximum-median estimated vertical TPU for a recently cleaned ATLAS Fansweep 20 bathymetric side-scan sonar, operating in optimal conditions. The median and maximum filter values are also shown (in black).

The vertical TPU estimated using the maximum-median method agrees well with the a-priori value calculated by the system for peak value and general trend. However, the estimated TPU captures the better performance of beams close to normal to the transducer face and also the regions of elevated uncertainty where the Fansweep 20 high and low frequency beams overlap.

As the sea floor is relatively consistent for the example shown in figure 3, increasing the along-track sampling window should not significantly increase the topography contribution, allowing greater measurement redundancy. 3-second along-track averaging for the same sonar is in figure 4.

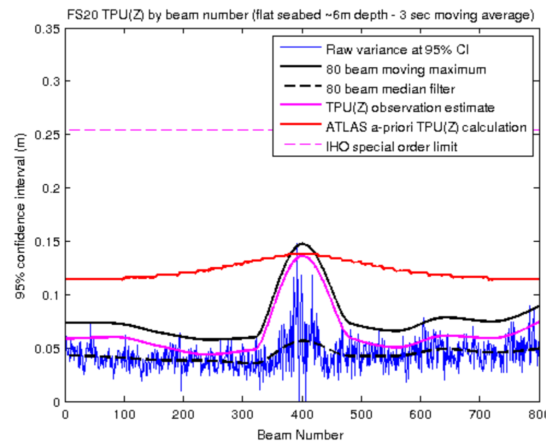

Figure 4: ATLAS FS20 a-priori calculated (red) and maximum-median estimated (magenta) uncertainty over a consistent sea-floor, 3 second along-track average, 80 beam across track filtered.

Again, the estimated peak uncertainty is matched by the ATLAS a-priori vertical TPU calculation. However, elsewhere across the swath, the a-priori calculation appears far more pessimistic than the vertical TPU estimated using the maximum-median method. This could have a significant effect on the creation of a CUBE surface. In the case of the a-priori vertical TPU, the center beams would be weighted too highly relative to the mid-swath beams. The generation and support of a CUBE hypothesis would then have an unrealistically high contribution from the most uncertain beams.

Using a different Fansweep 20 transducer, with a known systematic outer swath problem, produces the uncertainty result shown in figure 5.

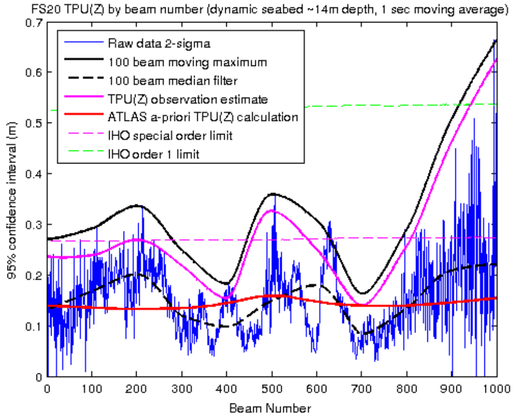

Figure 5: ATLAS FS20 a-priori (red) and maximum-median estimated (magenta) uncertainty. Systemic problem.

In this result, the a-priori vertical TPU estimate clearly fails to accurately show the true uncertainty. However, the TPU derived using the maximum-median method intrinsically captures the increased uncertainty due to the systemic problem.

In order to demonstrate the performance of the maximum-median method against a more traditional approach, figure 6 shows the uncertainty surface created by running the same survey line, six-times. Figure 7 shows a short along-track maximum-median result during one of those lines contributing to the figure 6 surface.

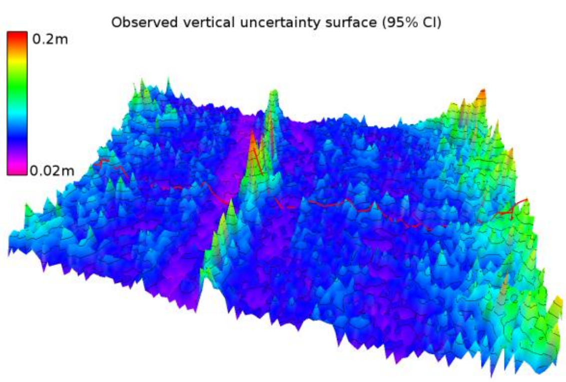

Figure 6: Uncertainty surface. ATLAS FS20, 600 beams, 600% coverage. Statistics from 6 runs of the same line.

So far, the maximum-median method for estimating vertical TPU has shown promising results when applied to the ATLAS Fansweep 20 bathymetric side-scan data. However, the goal of this process is to create realistic vertical TPU estimates in an architecture independent manner.

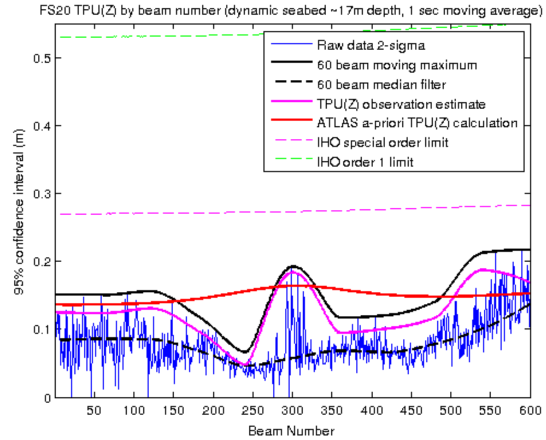

Figure 7: Maximum-median estimated vertical TPU using data contributing to the uncertainty surface in figure 6

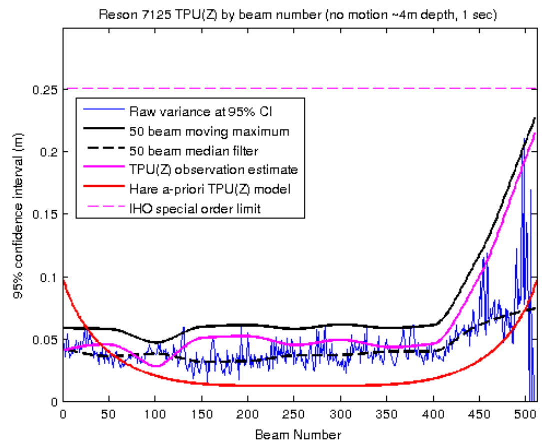

Figure 8: Reson 7125 calculated (red) and maximum-median estimated (magenta) TPU. Stationary vessel.

The same approach applied to a Reson 7125 beam-forming echo-sounder yields promising results in this respect. Figure 8 shows the vertical TPU calculated using the Hare[5] model compared with the maximum-median estimation. In this case, the data was collected on a stationary vessel so the simplified equation (i) was used to calculate vertical TPU for comparison.

At first glance, the maximum-median approach appears to give an unrealistic uncertainty estimate as there are stark differences between the a-priori calculated and maximum-median derived values.

However, when an entire Reson 7125 line is cross-checked against a 25cm CUBE generated using a different sounder (an Edgetech 4600), the variance per-beam of the Reson data matches the maximum-median estimate closely as shown in figure 9. Due to low quality flagging by the

Reson software, the beams from 446-512 were excluded from the cross-check.

Nevertheless, the data generally agrees with the maximum-median model, which suggests that the approach produces reasonable values independently of the sonar architecture.

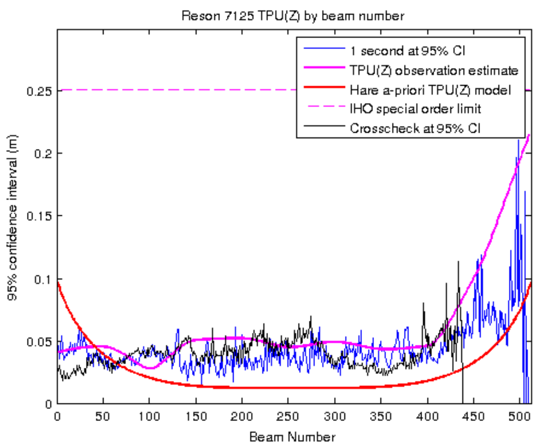

Figure 9: Comparison of maximum-median (magenta) and Hare (red) models with cross-check data (black) for the Reson 7125

4.2. CUBE surface comparison

The maximum-median method produces dynamic TPU values. In order to use dynamic TPU values with CARIS, they must be located in the raw files prior to processing. As ATLAS provides a C programming interface[13] to modify SURF format files, the Fansweep 20 data was chosen for CUBE surface comparisons.

Two 2m CUBE surfaces were generated using the data with the systematic problem shown in figure 5. The first CUBE used the a-priori modeled TPU and the second used the maximum-median derived vertical TPU value. Whilst most of the surface was unchanged, small areas in the center and on the outer-edges were significantly improved by the use of the maximum-median vertical TPU. The number of selected hypotheses influenced by the noisy outer-beams was significantly reduced in the second CUBE. This resulted in differences of up to 0.48m between the outer edges of the two CUBE surfaces. The differences are illustrated in figure 10.

The CUBE generated with the a-priori TPU also included more hypotheses of weaker strength per node in the center and outer edges. Most nodes had 3 or 4 weak hypotheses. Conversely, the CUBE generated using maximum-median estimated vertical TPU had very few nodes with multiple hypotheses. Those nodes that did still have multiple hypotheses, all had less than 3. The selected hypotheses were all of higher strength than the corresponding nodes in the a-priori TPU CUBE.

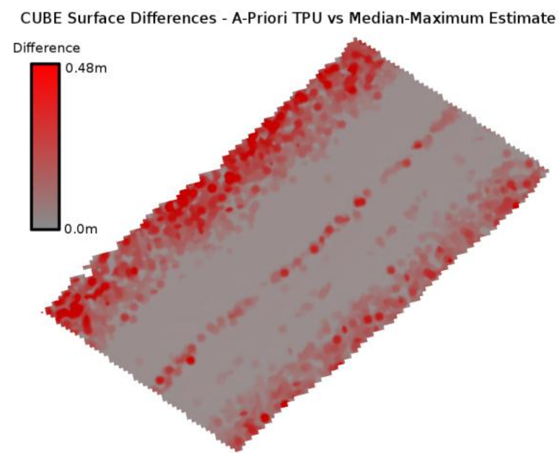

Figure 10: CUBE Surface Differences: a-priori - maximum-median derived vertical TPU

5. CONCLUSION

The maximum-median vertical TPU estimation method proposed in this paper shows promising results. Its utility in finding systematic data problems is demonstrated and it is shown to operate in an architecture independent manner. Tests also show that the maximum-median method may assist the CUBE algorithm in building a better surface when there is a mismatch between real sonar performance and a-priori TPU.

Using the maximum-median method exclusively would require further rigorous analysis of its intrinsic biases in estimating vertical TPU. Nevertheless, it can still be useful in aiding the CUBE algorithm to find the best estimate of likely depth. If it is employed in this fashion, any intrinsic error may be neglected by conducting traditional cross-line comparisons over the resulting CUBE surface. In any case, cross-line comparisons should be conducted to account for errors in the tide model that are not expressed by the maximum-median method.

Overall, the maximum-median method proposed in this paper has shown that it is useful in aiding the CUBE process in determination of likely depth. It can be employed in its present form, in conjunction with cross-line comparisons, for more reliable statistical cleaning of high density multi-beam survey data.

APPENDIX A

In order to calculate the median and maximum filters of variance, the following process was employed:

Consider a swath of *n* beams sampled for a set of pings, *p*. The variance of soundings by beam number for the set of pings is computed to produce a variance matrix of dimensions *(1, n):*

$$\sigma_T^2 \mid_p = \{\sigma_T^2(1) \mid_p, \sigma_T^2(2) \mid_p, \ldots \sigma_T^2(n) \mid_p\}$$

This matrix is then split into non-overlapping sub-matrices of dimensions *(1, n/w)*, with a window size, *w*, of say 10% of *n* beams.

This process decimates the data across the swath to a resolution of *n/w*, with each node containing *w* variance values. Each node now contains sufficient redundancy to build the median and maximum functions for that node.

The median function at the decimated resolution comprises of the median variance for each sub-matrix. Likewise, the maximum function at the decimated resolution comprises of the maximum variance for each sub-matrix. The swath position of these values is set as the center beam number of each sub-matrix.

For a sub-matrix beginning at *n=a*, the median and maximum functions give calculated values for beam number, B = *(2a+w)/2* (rounded to nearest whole beam number):

$$\vee(B) \mid_p = \vee\{\sigma_T^2(a) \mid_p, \sigma_T^2(a+1) \mid_p, \ldots \sigma_T^2(a+w) \mid_p\}$$

$$\tilde{x}(B) \mid_p = \tilde{x}\{\sigma_T^2(a) \mid_p, \sigma_T^2(a+1) \mid_p, \ldots \sigma_T^2(a+w) \mid_p\}$$

These values are then interpolated back to the resolution of the original data using cubic spline interpolation. The difference between the two functions is assumed to be the variance associated with the measurement by each beam, i.e.:

$$\sigma_B^2 \mid_p = \vee(\sigma_T^2 \mid_p) - \tilde{x}(\sigma_T^2 \mid_p)$$

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